Gravitational Collapse of a Quantum Field

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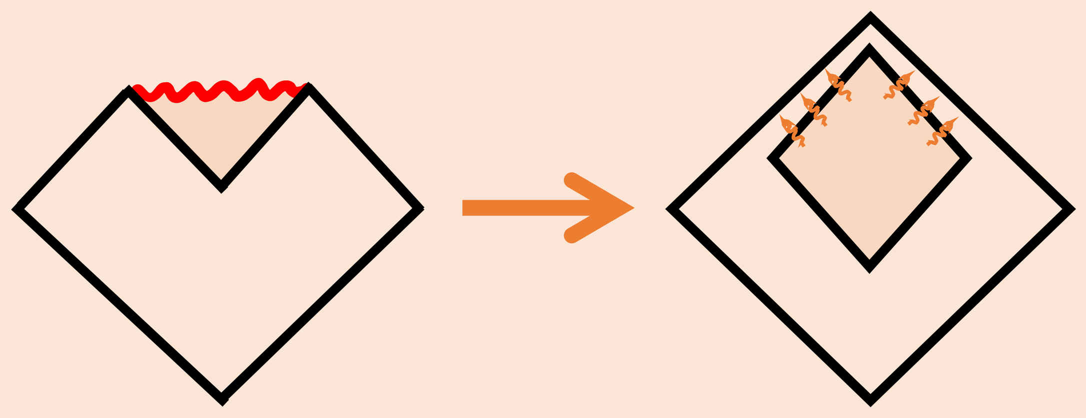
Classical vs Semiclassical

Black holes are among the most interesting objects in the Universe, and since Hawking's famous 1974 paper we know that they evaporate. This can be summarised in the Penrose diagrams below.



On the left the black hole is eternal, and one the right – once quantum effects are incorporated – the black hole evaporates and disappears completely. However, Hawking radiation – and other quantum effects – have only been studied approximately never in an exact sense (at least not in 4 dimensions). One way to study these would be to collapse a quantum field into a black hole and see what happens.

My project is to numerically simulate the collapse of a quantum field into a black hole and study the quantum effects.

Making a Quantum Field

It is notoriously difficult to simulate a quantum (scalar) field, since theoretically it is a sum of infinite quantum modes:

$$\hat{\Phi} = \sum_{l,m} \int dk \left[\hat{a}_{k,l,m} \tilde{u}_{k,l}(t,r) Y_l^m(\theta,\varphi) + \hat{a}^\dagger_{k,l,m} \tilde{u}^*_{k,l}(t,r) Y_l^{m*}(\theta,\varphi) \right]$$

However, using the notion of coherent states (special minimally uncertain states), one can separate the quantum field into a coherent state expectation value and additional quantum modes. The upshot is that the former is equivalent to a classical field, so that:

quantum = "classical" + "fluctuations"

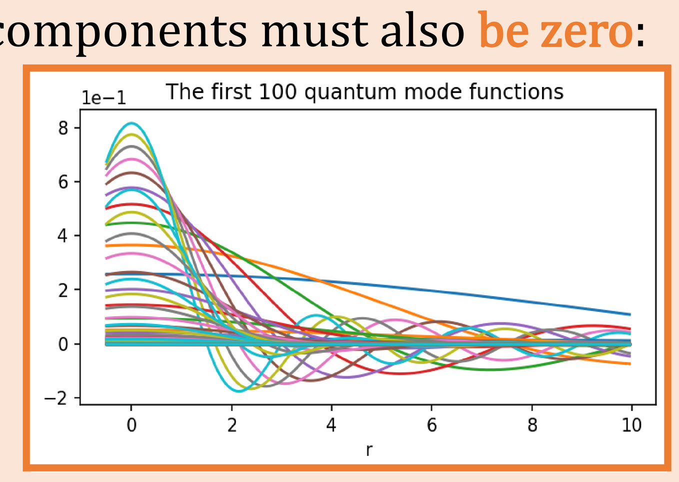
More concretely, the objects that appear in the stress-energy tensor of the quantum field have this same structure, for example:

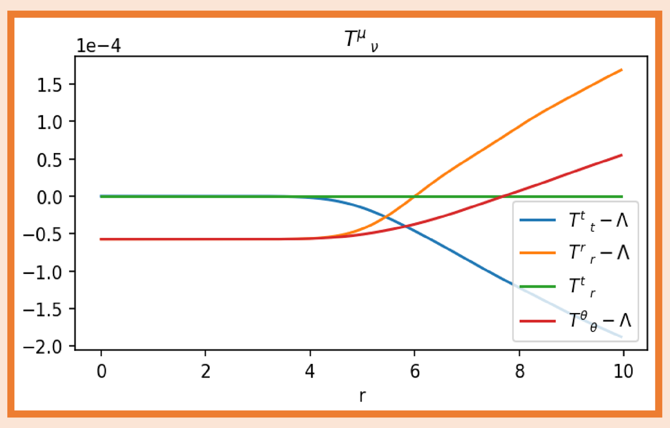
$$\langle\chi|\, \partial_t \hat{\Phi} \partial_t \hat{\Phi} \,|\chi\rangle = \partial_t \phi \partial_t \phi + \frac{\hbar c^2}{4\pi} \int dk \sum_l (2l+1) |\partial_t \tilde{u}_{k,l}|^2$$

Thus, one can use a simple classical field simulation and add some finite number of quantum modes!

Quantum Vacuum

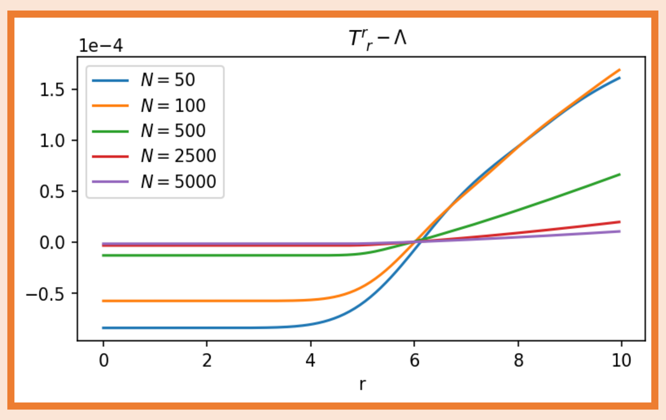
By virtue of the coherent state, one can try to build up a "quantum vacuum" using the quantum modes. Here vacuum means that the coherent state expectation value of the field is zero, therefore all the stress-energy tensor components must also be zero:





Adding up all those mode functions gives us a remarkably flat region!

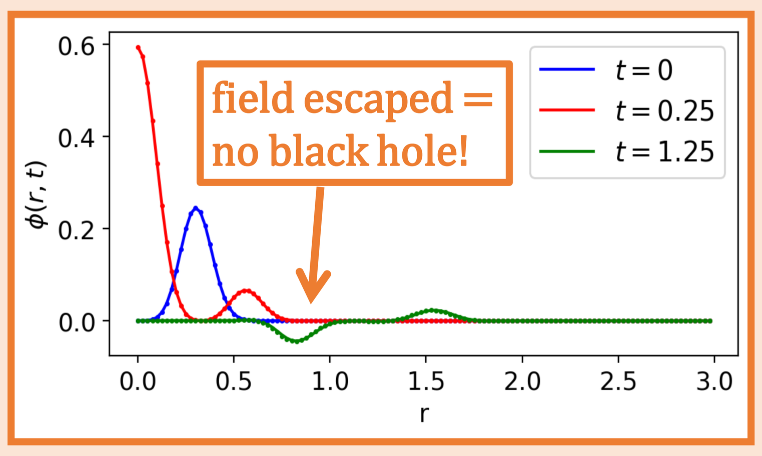
On the right, $T^r{}_r$ is plotted for different numbers of quantum modes. The more quantum modes we have, the stress tensor components are closer and closer to zero!

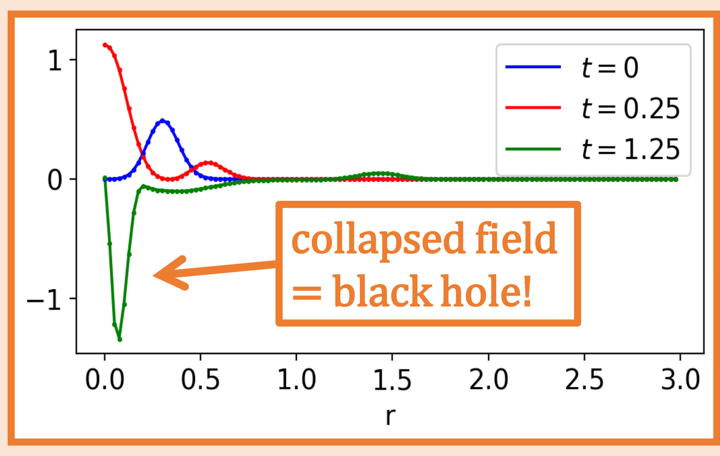


More modes, better vacuum!

The Simulation

As said before, the quantum field is built up using a coherent state expectation value (aka the "classical" part) and the mode functions. Here I will discuss the evolution of the former as a black hole forms / doesn't form. We start with an initial Gaussian lump at t=0, and evolve it in time:



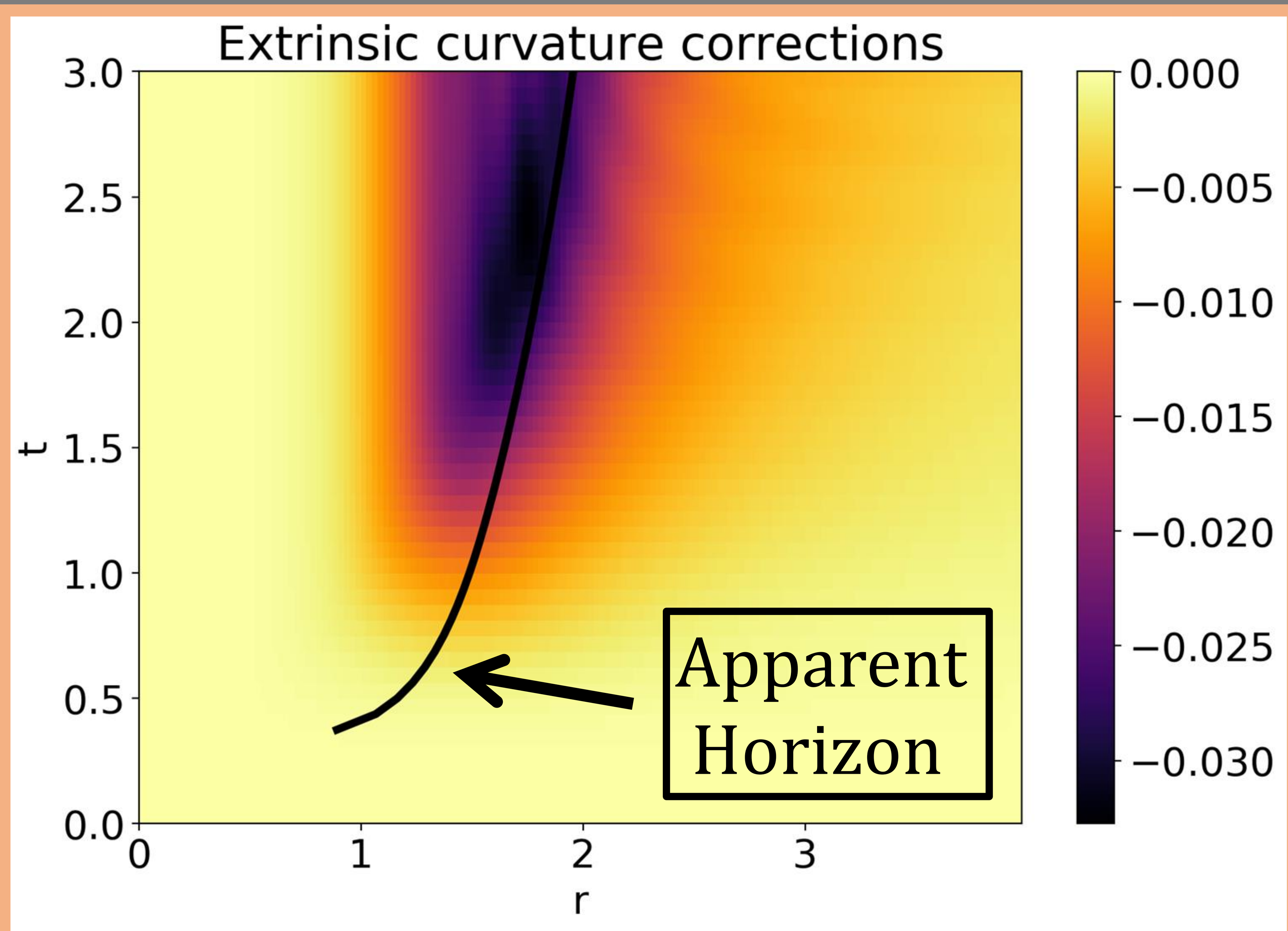


The lump splits into two smaller lumps moving away from each other. The lump moving towards the centre may or may not form a black hole, depending on its amplitude.

These are the expected evolutions from classical simulations, but one can check the difference between the stress-energy tensor components for the classical field and the quantum field after a black hole has formed. There is considerable difference that can be highlighted by looking at the backreaction on the spacetime by the quantum effects. This is plotted below!

However, what the quantum effects are exactly, we do not know yet.

Quantum Backreaction



On the plot above, the corrections due to the quantum mode functions on the extrinsic curvature is shown. This is essentially the backreaction of the quantum fluctuations on the spacetime.

Summary

Black holes evaporate, we know this from approximate calculations. We are solving the semiclassical – quantum field coupled to classical gravity – equations exactly using numerical simulations. We have managed to collapse a quantum field into a black hole, we see quantum effects, but do not know what they are exactly. For more information (including quantum Choptuik scaling) find our paper:

https://arxiv.org/abs/2111.11400